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# Quantum oscillations in the high frequency magnetoacoustic response of a quasi-two-dimensional metal

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## Abstract

In this work we present the results of theoretical analysis of magnetic quantum oscillations of the velocity and attenuation of high frequency ultrasound waves traveling in quasi-two-dimensional (Q2D) conductors. We chose a geometry where both the wavevector of the longitudinal sound wave and the external magnetic field are directed along the axis of symmetry of the Fermi surface. Assuming a moderately weak Fermi surface corrugation, we showed that the oscillating correction to the sound velocity may include a special term besides an ordinary contribution originating from quantum oscillations of the charge carrier density of states at the Fermi surface. This additional term is generated by a ‘phase stability’ resonance occurring when the charge carrier velocity in the direction of the wave propagation equals the sound velocity. The two oscillating contributions to the sound velocity are shown to differ in phase and shape, and they may have the same order of magnitude. The appearance of the extra term may bring about significant changes in magnetic quantum oscillations of the velocity of sound in Q2D conductors, especially at low temperatures.

(Some figures in this article are in colour only in the electronic version)

## 1. Introduction

It is common knowledge that quantization of the conduction electrons’ motion in strong magnetic fields gives rise to oscillations of the electron density of states (DOS) at the Fermi surface (FS) of a metal. These quantum oscillations generate several effects such as de Haas–van Alphen oscillations in the magnetization and Shubnikov–de Haas oscillations in the magnetoresistivity. The above effects were repeatedly used in studies of the FS geometries and other electronic properties of various conventional metals [1]. In the last three decades quasi-two-dimensional (Q2D) materials with metallic-type conductivity (intercalated compounds, organic metals and some others) attracted significant interest from the research community which resulted in extensive studies of their electronic characteristics. These materials reveal strong anisotropy of the electrical conductivity which reflects their layered structure. Conducting layers are rather weakly coupled to each other, so the charge carriers energy only

slightly depends on the momentum projection on the line perpendicular to the layers. Correspondingly, the Fermi surfaces of Q2D metals could be described as systems of weakly corrugated cylinders [2–4]. Again, magnetic quantum oscillations were widely used for obtaining important band-structure parameters. A theory of de Haas–van Alphen and Shubnikov–de Haas oscillations in Q2D conductors is proposed in several works [5–9].

So far, less attention has been paid to quantum oscillations in the elastic response of a Q2D metal to an external deformation generated by a traveling sound wave. Such oscillations in the sound velocity and attenuation are known in conventional metals. Studies of these oscillations were started by Gurevich *et al* [10] who first predicted the effect of giant quantum oscillations in the ultrasound attenuation; these studies were subsequently extended in several works [11–13]. Magnetic quantum oscillations in the sound velocity and attenuation were repeatedly observed in 3D metals [1].

It has been shown both theoretically and experimentally that the specific geometry of the Fermi surfaces of Q2D metals may cause significant differences in the size, shape and phase of the de Haas–van Alphen oscillations compared to those occurring in conventional metals. Therefore, one may expect similar features in the elastic response quantum oscillations in Q2D conductors to occur. The purpose of this work is to analyze these features. We concentrate on the case of high frequency ultrasound waves ( $\omega\tau > 1$  where  $\omega$  is the wave frequency and  $\tau$  is the scattering time for the charge carriers). The high frequency range is chosen because it provides opportunities for a richer and more complicated structure to be revealed in the oscillating corrections to the ultrasound velocity and attenuation.

## 2. Main equations and results

When a sound wave propagates through a metal, the crystalline lattice is periodically deformed, which brings changes to the electronic spectrum. These changes could be allowed for by introducing the deformation potential but here we omit them for brevity. Besides, the conduction electrons (charge carriers) affect the crystalline lattice by interaction with the self-consistent alternating electric field accompanying the sound wave as it travels in the metal. As a result, electron contributions appear in the expressions for elastic constants of the metal. When a strong external magnetic field is applied, these terms include corrections describing quantum oscillations in the elastic response of the metal to the sound wave.

To analyze the magnetic quantum oscillations in the elastic response of a Q2D metal we adopt the commonly used simple approximation for the charge carriers spectrum:

$$E(\mathbf{p}) = \frac{\mathbf{p}_\perp^2}{2m_\perp} - 2t \cos\left(\frac{\pi p_z}{p_0}\right). \quad (1)$$

Here, the  $z$  axis is taken to be at a right angle to the conducting layer plane,  $\mathbf{p}_\perp$  is the momentum projection in the layer plane, and  $m_\perp$  is the effective mass corresponding to the charge carriers motion in this plane. The parameter  $t$  in equation (1) is the interlayer transfer integral, and  $p_0 = \pi\hbar/d$  where  $d$  is the distance between the layers. When a quantizing magnetic field  $\mathbf{B}$  is applied perpendicularly to the layers, the Landau energy spectrum of the charge carriers has the form:

$$E_n^\sigma(p_z) = \hbar\Omega \left(n + \frac{1}{2}\right) + \frac{\sigma}{2}g\hbar\Omega_0 - 2t \cos\left(\frac{\pi p_z}{p_0}\right). \quad (2)$$

Here,  $\Omega$  is the cyclotron frequency,  $\Omega_0 = \beta B$ ,  $\beta$  is the Bohr magneton,  $\sigma$  is the spin quantum number and  $g$  is the spin splitting coefficient ( $g$ -factor).

We consider a longitudinal ultrasound wave traveling at a right angle to the conductivity layers (in parallel with the magnetic field) with frequency  $\omega$  and wavevector  $\mathbf{q} = (0, 0, q)$ . An expression for the wavevector of the sound wave can be written as follows:

$$q = \frac{\omega}{s} + \Delta q, \quad (3)$$

where  $s$  is the speed of sound in the absence of the external magnetic field, and  $\Delta q$  determines the magnetic field induced corrections to the velocity shift  $\Delta s$  and attenuation rate  $\Gamma$ :

$$\frac{\Delta q}{q} = \frac{\Delta s}{s} + \frac{i\Gamma}{2q}. \quad (4)$$

Using the general equations for the magnetoacoustic response of a metal [14, 15] we can obtain the following expression for the correction  $\Delta q$ :

$$\frac{\Delta q}{q} = -\frac{N^2}{2\rho_m s^2} \frac{1}{\eta} Y, \quad (5)$$

where  $\rho_m$  and  $N$  are the density of matter in the lattice and the charge carriers density, respectively, and  $\eta$  is the charge carriers DOS on the Fermi surface in the absence of the magnetic field. The function  $Y$  describing quantum oscillations of the elastic response has the form:

$$Y = \frac{1}{4\pi^2\hbar\lambda^2} \frac{1}{\eta} \times \sum_{n,\sigma} \int_{-\infty}^{\infty} dp_z \frac{f_n^\sigma(p_z) - f_n^\sigma(p_z - \hbar q)}{E_n^\sigma(p_z - \hbar q) - E_n^\sigma(p_z) + \hbar\omega + i\hbar/\tau}. \quad (6)$$

Here,  $f_n^\sigma(p_z)$  is the Fermi distribution function for the quasiparticles with the energies  $E_n^\sigma(p_z)$ , and  $\lambda$  is the magnetic length.

In further consideration, we assume, as usual, that the cyclotron quantum  $\hbar\Omega$  is small compared to the chemical potential of the charge carriers  $\mu$ . Then we employ the Poisson summation formula:

$$\sum_{n=0}^{\infty} \varphi\left(n + \frac{1}{2}\right) = \int_0^{\infty} \varphi(x) \left[1 + 2 \operatorname{Re} \sum_{r=1}^{\infty} (-1)^r \exp(2\pi i r x)\right]. \quad (7)$$

Using this formula we may present the function  $Y$  as a sum of a monotonic term  $Y_0$  and an oscillating correction  $\tilde{Y}$ . To proceed in computations of the oscillating function  $\tilde{Y}$  we change variables from  $n, p_z$  to  $E, p_z$ . Using these new variables, the integration over  $p_z$  is to be performed within finite limits determined by the minimum and maximum values of  $p_z$  at a given energy  $E$  ( $p_{\min}(E) < p_z < p_{\max}(E)$ ). These values are specified by the isoenergetic surfaces geometries within the Brillouin zone. Assuming that the frequency of the sound wave is not too high ( $\hbar\omega \ll m_\perp s^2$ ) we may expand the functions  $f_n^\sigma(p_z - \hbar q)$  and  $E_n^\sigma(p_z - \hbar q)$  in powers of  $\hbar q$  and approximate  $\tilde{Y}$  by the expression:

$$\tilde{Y} = -\frac{m_\perp}{2\pi^2\hbar^3} \frac{1}{\eta} \sum_{\sigma} \sum_{r=1}^{\infty} (-1)^r \int dE \frac{df^\sigma(E)}{dE} \times \int_{-p_0}^{p_0} dp_z \frac{v_z}{v_z - s - i/\tau} \cos\left[r \frac{\lambda^2}{\hbar^2} A_\sigma(E, p_z)\right], \quad (8)$$

where the longitudinal velocity  $v_z$  is given by

$$v_z = \frac{\partial E}{\partial p_z} = \frac{2\pi t}{p_0} \sin\left(\frac{\pi p_z}{p_0}\right) \quad (9)$$

and  $A_\sigma(E, p_z)$  is the cross-sectional area of the corresponding constant-energy surface. To properly estimate the integral over  $p_z$  in equation (8) we extend the integrand over the upper half of the complex plane, and we choose the integration path including the segment of the real part axis  $-p_0 \leq p_z \leq p_0$  and the circular arc with the radius  $p_0$ . The value of the integral crucially depends on the contribution from a pole at  $v_z = s + i/q\tau$  which may be situated within the integration contour.

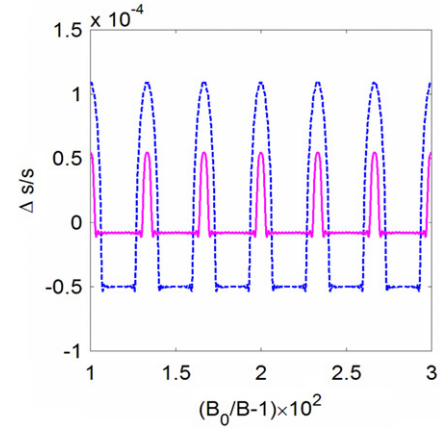
Strong anisotropy in the transport characteristics of Q2D conductors implies a pronounced difference between the quasiparticles velocities in the layer planes  $v_\perp$  and the longitudinal velocity  $v_z$ . Due to the smallness of the transfer parameter  $t$ , the longitudinal component of the charge carrier velocity on the Q2D Fermi surface may take on values significantly smaller than those typical for Fermi velocities in conventional metals. For a weakly warped Fermi surface ( $2\pi t/p_0 < s$ ),  $v_z$  is smaller than  $s$  and the integrand is an analytic function over the area within the integration path. Then the value of the of the integral over ' $p_z$ ' in equation (8) is solely determined by the contribution from the arc. Computing the latter we keep in mind that due to the relative smallness of the longitudinal velocity the ratio  $s/v_z$  in Q2D conductors may take on greater values, than in usual 3D metals, and we may expect an inequality  $\frac{s}{v_z} \sqrt{\mu/\hbar\Omega} > 1$  where  $\mu$  is the chemical potential of the charge carriers, to be satisfied when  $v_z$  takes on its maximum value  $v_0$ . Stipulating that  $\frac{s}{v_0} \sqrt{\mu/\hbar\Omega} > 1$  we may approximate  $\tilde{Y}$  as follows:

$$\tilde{Y} \approx -\frac{\alpha(ql)^2}{(1-i\omega\tau)^2} \sum_{r=1}^{\infty} (-1)^r D(r) \cos\left(\frac{2\pi r F}{B}\right) \times \left[ J_0\left(\frac{4\pi r t}{\hbar\Omega}\right) + J_2\left(\frac{4\pi r t}{\hbar\Omega}\right) \right]. \quad (10)$$

Here,  $\alpha = \frac{m_\perp p_0}{\pi^2} \frac{1}{\eta}$ ;  $l = v_0\tau$  is the mean free path of the charge carriers along the normal to the conducting layers;  $J_{0,2}(x)$  are the Bessel functions;  $F = cA/2\pi\hbar e$ ;  $A$  is the mean cross-sectional area of the Fermi surface. The damping factor  $D(r)$  includes the effects of temperature, scattering and spin splitting.

The obtained expression (10) describes magnetic quantum oscillations in the sound velocity and attenuation originating from the DOS oscillations in the strong magnetic fields. An explicit expression for the oscillating part of  $\Delta q$  may be written out by substituting equation (10) into (5). Oscillations in the velocity of sound described by equation (10) are shown in figure 1 assuming that the effects of temperature and scattering are moderately small. The oscillations look like a sequence of well distinguishable peaks separated by nearly flat regions. As the FS warping decreases, the peaks become sharper and lower in magnitude. We remark that the function  $\tilde{Y}$  as given by equation (10), goes to zero when the FS becomes purely cylindrical ( $t \rightarrow 0$ ). This is a reasonable result because at  $t = 0$  charge carriers cannot move between the conducting layers in the Q2D metal and, consequently, they cannot respond to the sound wave traveling across the latter.

Now, we consider the case of a more pronounced warping of the Fermi surface ( $2\pi t/p_0 > s$ ). Then the integrand in



**Figure 1.** Magnetic quantum oscillations of the velocity of a longitudinal ultrasound wave propagating along the symmetry axis of a weakly corrugated Fermi surface in a Q2D metal. The plotted curves are described by equations (5) and (10) assuming that  $F/B_0 = 300$ ,  $B_0 = 10$  T,  $t/\hbar\Omega(B_0) = 0.1$  (dashed line) and 0.04 (solid line).

the integral over ' $p_z$ ' in equation (8) has the pole within the contour of integration at  $p_z = s + i/q\tau$ . Correspondingly, the oscillating function  $\tilde{Y}$  equals the sum of the residue from the pole  $\tilde{Y}_1$  and the contribution from integration over the arc  $\tilde{Y}_2$ . The term  $\tilde{Y}_1$  can be written in the form:

$$\tilde{Y}_1 \approx -\frac{\alpha\pi i}{2} \frac{s}{v_0} \sum_{r=-\infty}^{\infty} (-1)^r D(r) \times \exp\left(-2\pi i r \frac{F}{B}\right) \cos\left(\frac{4\pi r t}{\hbar\Omega} \sqrt{1 - \frac{s^2}{v_0^2}}\right). \quad (11)$$

The expression for the remaining term  $\tilde{Y}_2$  depends on the value taken by  $\frac{s}{v_0} \sqrt{\mu/\hbar\Omega}$ . Assuming  $\frac{s}{v_0} \sqrt{\mu/\hbar\Omega} > 1$  we obtain  $\tilde{Y}_2$  in the form closely resembling DOS oscillations in a quantizing field:

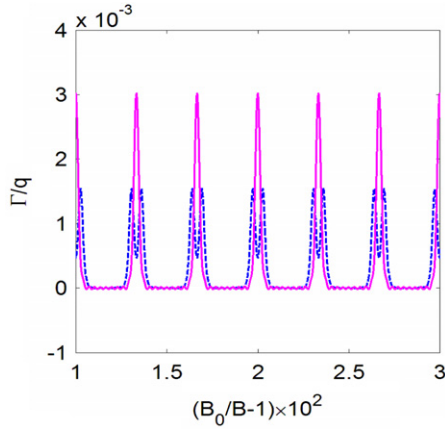
$$\tilde{Y}_2 \approx \alpha \sum_{r=1}^{\infty} (-1)^r D(r) J_0\left(\frac{4\pi r t}{\hbar\Omega}\right) \cos\left(2\pi r \frac{F}{B}\right). \quad (12)$$

For a noticeably warped FS ( $4\pi t/\hbar\Omega > 1$ ) we may appropriately approximate the Bessel function and present  $\tilde{Y}_2$  in the form resembling the expression suitable for 3D metals:

$$\tilde{Y}_2 = \sqrt{\frac{\hbar\Omega}{8\pi^2 t}} \sum_{r=1}^{\infty} \frac{(-1)^r}{\sqrt{r}} D(r) \left[ \cos\left(\frac{2\pi r F_{\min}}{B} + \frac{\pi}{4}\right) + \cos\left(\frac{2\pi r F_{\max}}{B} - \frac{\pi}{4}\right) \right], \quad (13)$$

where  $F_{\min}$  and  $F_{\max}$  are associated with the FS minimum and maximum cross-sectional areas  $A_{\min}$  and  $A_{\max}$ , respectively. The origin of the term  $\tilde{Y}_1$  is rather more complicated, and its occurrence needs some particular explanation.

As follows from the form of the charge carriers spectrum in a strong magnetic field given by equation (2), the quasiparticles energies corresponding to their motions in the planes perpendicular to the magnetic field are quantized. Therefore, at a certain magnitude of  $\mathbf{B}$  the longitudinal part of

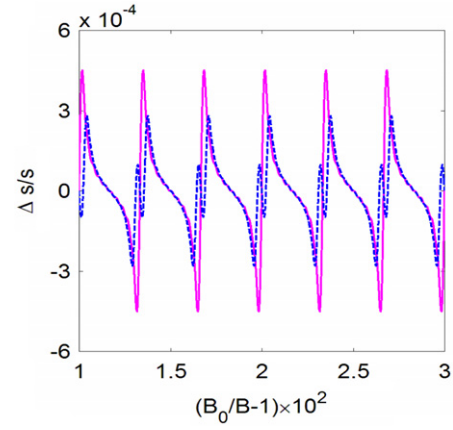


**Figure 2.** Quantum oscillations in the attenuation of the longitudinal high frequency sound propagating along the symmetry axis of a moderately corrugated Fermi surface in a Q2D metal. The curves are plotted assuming that  $F/B_0 = 300$ ,  $B_0 = 10$  T and  $t/\hbar\Omega(B_0) = 4$ .

a quasiparticle energy at the Fermi surface can take on values belonging to a set of intervals with widths of the order of the thermal energy  $kT$  ( $k$  is Boltzmann’s constant). This leads to a similar distribution of possible values of the longitudinal velocity  $v_z = \partial E/\partial p_z$  at the Fermi surface. Within the low temperature limit ( $T \rightarrow 0$ ) the intervals are reduced to points, so we obtain a discrete set of possible  $v_z$  values for every magnitude of the field  $\mathbf{B}$ . Quasiparticles on the Fermi surface may absorb phonons provided that the conservation laws for both energy and momentum are satisfied. This occurs when the longitudinal velocity  $v_z$  coincides with the speed of sound  $s$ . At certain magnitudes of the magnetic field we may find  $v_z = s$  among the allowed values of the longitudinal velocity, so the relevant quasiparticles may absorb the phonons. This happens regardless of the value of the electron velocity component  $\mathbf{v}_\perp$  perpendicular to the ultrasound wavevector  $\mathbf{q}$ . To understand this situation one may recall a semiclassical Landau damping mechanism providing the effective absorption of the energy of electric field associated with the sound wave by electrons moving ‘in phase’ with the wave. The ‘in phase’ motion occurs when the electron velocity component along the wave coincides with the speed of sound because in this case the electron is consistently seeing the same electric field irrespectively to the velocity component  $\mathbf{v}_\perp$  which is associated with the electron motion in parallel with the wavefront.

A slight change in  $B$  shifts all of the allowed values of  $v_z$  and leads to a situation where there are no charge carriers at the Fermi surface capable of phonon absorption. Varying the magnetic field magnitude we successively satisfy and destroy the ‘resonance’ conditions for the phonon absorption generating the giant quantum oscillations in the ultrasound attenuation. Such oscillations are described by the imaginary part of the function  $\tilde{Y}_1$  which could be rewritten in the form:

$$\text{Im } \tilde{Y}_1 = -\frac{\alpha\pi}{4} \frac{s}{v_0} \sum_{r=-\infty}^{\infty} (-1)^r D(r) \times \left[ \cos\left(\frac{2\pi r F_+}{B}\right) + \cos\left(\frac{2\pi r F_-}{B}\right) \right]. \quad (14)$$



**Figure 3.** Oscillating corrections to the velocity of a longitudinal sound wave traveling along the symmetry axis of a moderately corrugated FS of a Q2D metal originating from the ‘phase stability’ resonance in a quantizing magnetic field. The plotted curves are described by equations (5) and (15) for  $F/B_0 = 300$ ,  $B_0 = 10$  T,  $t/\hbar\Omega(B_0) = 4$ .

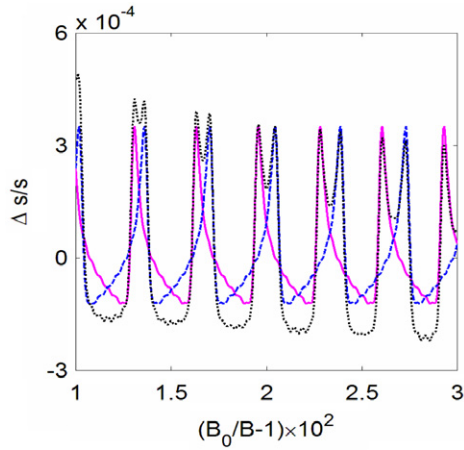
Here,  $F_\pm = cA_\pm/2\pi\hbar e$ ,  $A_\pm = A \pm \delta A$ , and  $\delta A = 4\pi t m_\perp \sqrt{1 - s^2/v_0^2}$ . This expression agrees with the well known result of [10] theoretically describing so called giant quantum oscillations in the ultrasound attenuation in 3D metals. Sound attenuation oscillations in a Q2D conductor display a sequence of very sharp peaks separated by regions of much weaker attenuation resembling those observed in conventional 3D metals. This is shown in figure 2. The attenuation peaks are split in two, as presented in the figure. This happens because the charge carriers belonging to the FS cross-sections with slightly different areas  $A_+$  and  $A_-$  strongly contribute to the ultrasound attenuation which is the effect of the Q2D Fermi surface geometry. We remark that these ‘efficient’ cross-sections do not coincide with those possessing minimum or maximum areas. Omission of the correction  $\delta A$  results in disappearance of the splitting (see the solid line in the figure 2).

Besides the imaginary part, the function  $\tilde{Y}_1$  has a nonzero real part accounting for an extra term in the oscillating correction to the sound velocity. As follows from equation (11):

$$\text{Re } \tilde{Y}_1 = \frac{\alpha\pi}{4} \frac{s}{v_0} \sum_{r=-\infty}^{\infty} (-1)^r D(r) \times \left[ \sin\left(\frac{2\pi r F_+}{B}\right) + \sin\left(\frac{2\pi r F_-}{B}\right) \right]. \quad (15)$$

This correction describes quantum oscillations controlled by the same mechanism as the giant quantum oscillations in the ultrasound attenuation. The magnetic field dependencies of the above oscillating terms are presented in figure 3. We see that both shapes and phases of the oscillations described by  $\text{Re } \tilde{Y}_1$  (figure 3) and  $\tilde{Y}_2$  (figure 4) differ. Again, the profile of the solid line in figure 3, results from contributions from the FS cross-sections overlapping with areas  $A_+$  and  $A_-$ , respectively. Neglecting the correction  $\delta A$  one obtains a simpler shape of the oscillations shown as the solid line. A rather complicated





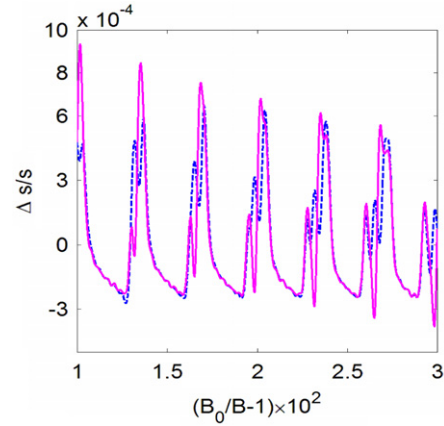
**Figure 4.** Quantum oscillating corrections to the velocity of a longitudinal sound wave traveling along the symmetry axis of a moderately corrugated Fermi surface of a Q2D metal originating from charge carriers DOS oscillations in a strong magnetic field. The curves are plotted using by equations (5) and (13). The relevant parameters take on the same values as in the previous figure.

pattern of the DOS related oscillations also originates from the Fermi surface rippling. Actually, the solid line in figure 4 appears as a superposition of contributions from the FS cross-sections with minimum and maximum areas. The resulting dependence of the oscillating correction to the speed of sound wave including both kinds of corrections is shown in figure 5.

Now, we briefly discuss possibilities for an experimental observation of the predicted features in magnetic quantum oscillations of high frequency ultrasound velocity and attenuation in Q2D conductors. We concentrate on the organic metals for FSs of these materials were studied in numerous experiments [3, 4]. To estimate the order of  $v_0$  we assume  $A = 1.31 \times 10^{-49} \text{ (kg m s}^{-1}\text{)}^2$ ,  $m_{\perp} = 4.5m_0$  ( $m_0$  being the free electron mass) and  $(A_{\max} - A_{\min})/A \sim 0.04$  which is close to the values reported for a typical Q2D organic metal  $\beta - (ET)_2\text{IBr}_2$  [4]. Using these data we may estimate  $v_0$  within the adopted model of the charge carriers spectrum given by equation (1), and we get  $v_0 \sim 4 \times 10^3 \text{ m s}^{-1}$ , and  $t/\hbar\Omega \sim 10$  at  $B = 10 \text{ T}$ . The obtained approximate value for the longitudinal velocity is far less than typical values of the Fermi velocity but no less than the speed of sound in conventional 3D metals. However, the speed of ultrasound wave propagating along the FS symmetry axis in a Q2D organic metal is smaller than in 3D metals due to the weak connection between adjacent conducting layers. This gives grounds to believe that the condition  $v_0 < s$  may be satisfied in organic metals, and the effects discussed in this work could be available for experimental observation. One could expect these effects to occur at high frequencies of the ultrasound ( $ql > v_0/s$ ) and at low temperatures.

### 3. Conclusion

In conclusion, in this work we theoretically analyzed magnetic quantum oscillations in the elastic response of a Q2D metal to a high frequency ultrasound wave. Within a chosen geometry a



**Figure 5.** Superposition of the quantum oscillations described by equations (5), (10) and (11). The relevant parameters take on the same values as in the figures 3 and 4.

longitudinal ultrasound wave was assumed to travel along the magnetic field directed in parallel with the axis of symmetry of the Fermi surface. We showed that both sound velocity and attenuation reveal quantum oscillations provided that the FS corrugation is not too weak. The case of moderately strong Fermi surface warping ( $t > sp_0$ ) is especially interesting. We showed that in this case the oscillating correction to the sound velocity includes two terms of different origins and this is the main result of this work. At low temperatures ( $2\pi^2kT < \hbar\Omega$ ) the two terms significantly differ in shape and phase. Also, their periods are slightly different, as follows from equations (11)–(15). An extra term in the oscillating correction to the velocity of sound proportional to the function  $\tilde{Y}_1$  appears due to some kind of ‘phase stability’ resonance occurring when a charge carrier moves along the magnetic field at the same velocity as the sound wave travels. Therefore, the relevant quasiparticles are seeing the same phase of the wave all the while between consecutive collisions, and can efficiently absorb phonons. So, the above oscillations in the sound velocity appear due to the same reason as giant quantum oscillations in the sound attenuation studied in 3D metals. Overall, the present results enable us to better understand specific features of magnetic quantum oscillations in the elastic response of Q2D conductors, and they could be easily generalized to describe 3D metals. We remark that the inequality  $\frac{s}{v_0}\sqrt{\mu/\hbar\Omega} > 1$  is probably violated in good metals. Therefore, the expressions for the oscillating corrections to the sound velocity unrelated to ‘phase stability’ resonance may not closely follow in shape and phase the DOS quantum oscillations. We discuss these issues in the appendix.

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## Appendix

Expressions for the oscillating terms in the function  $Y$  appropriate for a conventional 3D metal are derived in earlier works (see [15]). Here, we give these expressions without derivation assuming for simplicity that the FS possesses a spherical shape. Then equation (8) takes on the form:

$$\begin{aligned} \tilde{Y} = & \frac{1}{2} \sum_{\sigma} \sum_{r=1}^{\infty} \int dE \int_{-p_{\max}(E)}^{p_{\max}(E)} dp_z \\ & \times \frac{f^{\sigma}(E, p_z) - f^{\sigma}(E, p_z - \hbar q)}{E_{\sigma}(p_z - \hbar q) - E_{\sigma}(p_z) + \hbar\omega + i/\hbar\tau} \\ & \times \cos\left(r \frac{\lambda^2}{\hbar^2} A_{\sigma}(E, p_z)\right). \end{aligned} \quad (16)$$

Here,  $p_{\max}(E)$  is the maximum value of the longitudinal momentum  $p_z$  at a given energy  $E$ .

Assuming  $\gamma = \sqrt{2\mu/\hbar\Omega} \gg 1$ , and taking into account the main terms in the expansion of the function  $Y$  in powers of the small parameter  $\gamma^{-1}$  we obtain:

$$\begin{aligned} \tilde{Y}_1 = & \frac{1}{2qR} \sum_{r=1}^{\infty} \frac{(-1)^r}{r} D(r) \\ & \times \left\{ \exp\left[-\pi i r \gamma^2 \left(1 - \left[u + \frac{\hbar q}{2p_F}\right]^2\right)\right] \right. \\ & \left. + \frac{u - \frac{\hbar q}{2p_F}}{\left|u - \frac{\hbar q}{2p_F}\right|} \exp\left[-\pi i r \gamma^2 \left(1 - \left[u - \frac{\hbar q}{2p_F}\right]^2\right)\right] \right\}, \end{aligned} \quad (17)$$

where  $u = v_z/v_F$ ,  $R$  is the cyclotron radius and  $v_F$ ,  $p_F$  are the Fermi velocity and Fermi momentum for electrons, respectively. The correction  $\tilde{Y}_2$  may be written in the form:

$$\begin{aligned} \tilde{Y}_2 = & \frac{1}{2\gamma} \sum_{r=1}^{\infty} \frac{(-1)^r}{\sqrt{r}} D(r) \left\{ \exp\left(ir\gamma^2 - i\frac{\pi}{4}\right) G_r^- \right. \\ & \left. + \exp\left(-ir\gamma^2 + i\frac{\pi}{4}\right) G_r^+ \right\} \end{aligned} \quad (18)$$

where

$$G_r^{\pm} = \pm \frac{i}{2} \int_0^{\infty} \exp(\pm iy) \exp\left(-\sqrt{2\pi r \gamma^2 u^2 y}\right) dy. \quad (19)$$

When the frequency of the sound wave is not too high we may expect the ratio  $\hbar q/p_F$  to take on values much smaller than  $s/v_F$ . Under this assumption we may expand the exponents in the equation (17) in powers of a small parameter  $\hbar\omega/m_{\perp}s^2$ , and we get

$$\text{Re } \tilde{Y}_1 = \frac{\pi u}{4} \sum_{r=-\infty}^{\infty} (-1)^r D(r) \sin\left[\frac{2\pi r F_{\max}}{B} \left(1 - \frac{\overline{\delta A}}{A_{\max}}\right)\right] \quad (20)$$

$$\begin{aligned} \text{Im } \tilde{Y}_1 = & -\frac{\pi u}{4} \\ & \times \sum_{r=-\infty}^{\infty} (-1)^r D(r) \cos\left[\frac{2\pi r F_{\max}}{B} \left(1 - \frac{\overline{\delta A}}{A_{\max}}\right)\right]. \end{aligned} \quad (21)$$

Here,  $\overline{\delta A}$  is the difference between  $A_{\max}$  and the cross-sectional area corresponding to the ‘effective’ FS cross-section

where the charge carriers with  $v_z = s$  do belong. In the considered case of a spherical FS the correction  $\overline{\delta A}/A_{\max} = u^2$ . To compare equations (14) and (15) with the presented results appropriate for a 3D metal, we rewrite the expressions for  $A_{\pm}$  as follows:

$$F_+ = A_{\min} \left(1 + \frac{\delta A}{A_{\min}}\right), \quad (22)$$

$$F_- = A_{\max} \left(1 - \frac{\delta A}{A_{\max}}\right), \quad (23)$$

where  $\overline{\delta A} = 4\pi t m_{\perp} (1 - \sqrt{1 - s^2/v_0^2})$ . Substituting these expressions into equations (14) and (15) we see that the terms including  $F_-$  have the form similar to that given by equations (20) and (21). The resemblance becomes closer assuming that  $s/v_0 \ll 1$  as it occurs in regular metals. In this case  $\overline{\delta A} \approx 2\pi m_{\perp} t s^2/v_0^2$ . For more realistic models of a 3D metal when FS possesses cross-sections with both maximum and minimum areas the term similar to that including  $F_-$  will appear in equations (20) and (21), as well. It is worthwhile to mention that similarity in the expressions for the correction  $\tilde{Y}_1$  appropriate for 3D and Q2D conductors occurs irrespective of the value of  $\frac{s}{v_F} \sqrt{\mu/\hbar\Omega}$ .

To compare the expressions (18) and (19) with the corresponding result for Q2D conductors given by equation (13), one must keep in mind that equation (13) is derived assuming  $\frac{s}{v_0} \sqrt{\mu/\hbar\Omega} > 1$ . Under the similar assumption  $\gamma u > 1$  one may approximate the functions  $G^{\pm}$  using asymptotic expressions for Fresnel integrals and get  $G^{\pm} \approx 1$ . So, we have

$$\tilde{Y}_2 = \frac{1}{\gamma} \sum_{r=1}^{\infty} \frac{(-1)^r}{\sqrt{r}} D(r) \cos\left(\frac{2\pi r F_{\max}}{B} - \frac{\pi}{4}\right). \quad (24)$$

We see that the main difference between Q2D and 3D conductors is given by the small factor ( $\sqrt{\hbar\Omega}/8\pi^2 t$  and  $\gamma^{-1}$ , respectively) determining the order of oscillations magnitudes. To illustrate the transformation from the Q2D to the 3D case we may write

$$\sqrt{\frac{\hbar\Omega}{8\pi^2 t}} = \frac{1}{\pi\gamma} \sqrt{\frac{A}{A_{\max} - A_{\min}}}. \quad (25)$$

When the FS corrugation becomes significant, so that  $A_{\max} - A_{\min} \sim A$ , we could treat the considered conductor as a 3D metal with the corresponding order of magnitude of magnetic quantum oscillations of charge carriers DOS, and related to DOS characteristics. We remark that the inequality  $(s/v_0) \sqrt{\mu/\hbar\Omega} > 1$  hardly could be satisfied in good metals with nearly spherical Fermi surfaces. However, oscillating terms of the form (24) could appear in the magnetoacoustic response of a 3D metal as contributions from small segments of a complicated FS including several sheets.

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